SPACE DOMAIN MODE-MATCHING METHOD (SDM^3) -- AN EFFICIENT TECHNIQUE FOR COMPLEX MICROWAVE INTEGRATED CIRCUITS

Ke WU, Yansheng XU, and Renato G. BOSISIO

Groupe de Recherches Avancées en Microondes et en Électronique Spatiale
École Polytechnique, C. P. 6079, Succ. "A", Montréal, Canada H3C 3A7

ABSTRACT

A novel approach named "Space-Domain Mode-Matching Method (SDM^3)" is proposed to analyze complex microwave integrated circuits. This technique is based on the method of lines with vertical space discretization and conceptual Discrete Fourier Transforms (DFT) in multisubregions. This feature is exploited to improve the simulation related to the edge singularities on planar conductors. This novel approach is able to accurately model various bilaterally unbounded microwave circuits including composite, finite-width substrates and segmented topology. Numerical results are shown for both quasi-static and hybrid-mode analysis.

INTRODUCTION

It has been well recognized that numerical approaches serve as the base for state-of-the-art computer aided design (CAD) for various complex hybrid and monolithic microwave integrated circuits including millimeter-wave structures. In general, extensive use of most efficient CAD tools available today are mainly limited by ever-increasing demand for highly accurate prediction of realist circuitry over large frequency range. On the other hand, these modeling are problematic and even not applicable to some frequently encountered structures such as composite dielectric with finite width of layers, and segmented waveguide components. In addition, exact simulation of bilaterally unbounded cross-section for a number of waveguides is essential in determining possible surface- and/or leaky-wave propagation. Moreover, some relatively accurate techniques are not able to provide fast convergence due to involvement of edge singularities over metallic conductors. Although general-purpose numerical methods developed so far present an attractive feature of versatile applications for a wide range of microwave circuits with arbitrary shapes, they are at the expense of long CPU time and large RAM requirement. Besides, exact unbounded boundaries are difficult to be handled in these techniques, which usually employ complete discretization of structures in space.

In order to tackle these problems, an efficient technique named "Space-Domain Mode-Matching Method (SDM^3)" is proposed based on the modeling principle of the method of lines. This novel technique highlights the application of discrete Fourier transforms in different subregions. It retains the advantage of the conventional method of lines while eliminates some of its limitations. The discretization scheme is also different from that of the conventional method. In addition, this approach is able to accurately simulate composite structures with finite width of layers and segmented dielectric.

THEORY

The procedure used in the analysis of microwave integrated circuits by SDM^3 consists of the following steps:

A. Divide whole structure into appropriate subregions with different scheme of the space discretization.
B. Apply the method of lines (Discrete Fourier Transform - DFT) in each subregion independently and make field matching at the interface between adjacent subregions in the space domain.
C. Obtain recursive relationship via cascaded field/potential matrices and transform boundary conditions from one interface to another.
D. Derive a deterministic potential equation or a determinantal eigenvalue equation for the quasi-static or hybrid-mode analysis.

The theoretical framework of the proposed technique is demonstrated through an example of composite multilayer microstrip line with a bilaterally unbounded cross-section. As shown in Fig.1, the entire structure is divided into five subregions in which the region III consists of two parts denoted III' and III". On the other hand, a discretization scheme is used such that discrete potential lines are parallel to two ground planes and the microstrip. As a result, the line discretization of unknown charge or current density on the metallic strip, which is usually done in the method of lines, is effectively avoided. Therefore, numerical error due to the edge singularity can be greatly reduced. This is in particular useful in analyzing simultaneously wide and narrow strips for which the discretization size can remain to be constant with consistent accuracy. Moreover, the unbounded bilateral transverse section can be exactly simulated without resorting to any artificial absorbing boundary conditions. This is

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because extent of the discrete lines is able to be infinite along ± y-direction in the method of lines. This is in particular interesting for determining physical characteristics of possible surface- and/or leaky-wave propagation.

The theoretical background of the method of lines is simply based on concept of the discrete Fourier transform [1]. In fact, such segmentation and discretization of the structure can be regarded as space-domain eigen-field expansion in relevant subregions. This nature can also be interpreted as the space-domain counterpart of continuous Fourier spectral expansion in related entire subregion. The characteristic equation of the structure is obtained through subsequent field matching at interfaces of these subregions in the original space-domain. This is why this technique is called space-domain mode-matching method (SDM[3]).

I. Quasi-Static Analysis

Major steps of analysis are demonstrated by solving the Laplace equation $(\nabla \cdot (\varepsilon \nabla \psi) = 0)$. This equation can be transformed in each subregion by diagonalizing the resulting difference matrices from the second-order differential operator in the discrete space-domain,

$$\frac{d^2\vec{\phi}_i}{dy^2} - \overline{\kappa}_i^2 \cdot \vec{\phi}_i = 0 \qquad (i = I, II, IV, V)$$

$$\frac{d^2\vec{\phi}_j}{dy^2} - \overline{\kappa}_j^2 \cdot \vec{\phi}_j = \vec{p}_j \qquad (j = III', III'') \qquad (1)$$

where $\overline{\kappa}$ is a diagonal matrix, $\vec{\phi} = T \cdot \vec{\psi}$ (T is the matrix of transform defined in each subregion) and the vector $\vec{p}$ presents the source due to non-zero potential on the microstrip. Telegraph-type solution of (1) and use of the boundary conditions at interfaces between two adjacent subregions in the original domain lead to

$$\left.\frac{d\vec{\psi}_{III}}{dy}\right|_{y=-w} = \overline{H} \cdot \vec{\psi}_{III}\Big|_{y=-w} \qquad (2)$$

In (2), the infinite extent of the transverse cross-section is implicitly involved. According to the geometry of the model to be considered, (2) should be splitted into three parts and a matrix equation on the boundary of subregions II-III (y = -w) is derived as

$$\begin{bmatrix} \frac{d\vec{\psi}_{III'}}{dy} \\ \frac{d\psi_0}{dy} \\ \frac{d\vec{\psi}_{III''}}{dy} \end{bmatrix} = \begin{bmatrix} H_{11}^{(N_1 \times N_1)} & H_{12}^{(N_1 \times 1)} & H_{13}^{(N_1 \times N_2)} \\ H_{21}^{(1 \times N_1)} & H_{22}^{(1 \times 1)} & H_{23}^{(1 \times N_2)} \\ H_{31}^{(N_2 \times N_1)} & H_{32}^{(N_2 \times 1)} & H_{33}^{(N_2 \times N_2)} \end{bmatrix} \cdot \begin{bmatrix} \vec{\psi}_{III'} \\ \psi_0 \\ \vec{\psi}_{III''} \end{bmatrix} \qquad (3)$$

in which the dimensions N_1 and N_2 are determined by the line number of discretization in the regions III' and III'', respectively. Similar relationship can readily be obtained at the boundary of y = 0. The potential on the strip ψ_0 is set to be a constant u_0.

After some mathematical manipulations, it is easy to derive the matrix relationship between the transformed potentials and their y-dependent derivatives at y = -w and y = 0. Transforming back into the original domain, a deterministic matrix of potential can be obtained and the potential on the N_1^{th} line of the region III' (the line next to the strip) and the 1st line of the region III'' can then be determined.

The charge density along the strip q(y) is then calculated and the capacitance of the structure is found from the following equation:

$$C = \frac{\int_{-w}^{0} q(y) \cdot dy}{u_0} \qquad (4)$$

Note that the total charge can be calculated through simple integration or numerical summation. It is seen at this point that numerical accuracy is maintained the same for wide strips as for narrow strips. However, this is not the case in the conventional method of lines. On the other hand, the novel technique is able to provide a very accurate description of the subtle edge singularity and charge density profile over the strip. Next, the effective dielectric constant and characteristic impedance of the line can easily be obtained.

II. Hybrid-Mode Analysis

The procedure to be used in the hybrid-mode analysis is similar to the quasi-static case except the handling of subregions involving metallic strips. For simplicity, the same structure as Fig.1 with symmetrical features (i.e., $s_1 = s_2 = s$ and $\varepsilon_1 = \varepsilon_3$) is considered in the following analysis. It is evident that this restriction may easily be removed and such an analysis can be extended to generalized cases. Since the electric or magnetic wall can be placed at the middle of the structure, it is enough to handle four subregions denoted I, II, III' and III''.

Assuming that the circuit is lossless with a strip of vanishing thickness, electromagnetic potentials ψ^e (LSM-x) and ψ^h (LSE-x) are discretized through the conventional way for each region. In the homogeneous region (III'), field quantities in the transformed domain can be formulated in terms of both potentials:

$$T_e^t \cdot \overline{r}_e^{-1} \cdot \vec{E}_x = \tilde{E}_x = \frac{\beta^2 - \gamma_e^2}{j\omega\varepsilon_0} \vec{\phi}^e$$

$$T_h^t \cdot \overline{r}_h^{-1} \cdot \vec{E}_z = \tilde{E}_z = \frac{\beta\delta^t}{\omega\varepsilon_0 h_0} \vec{\phi}^e + \frac{d\vec{\phi}^h}{dy}$$

$$T_h^t \cdot \overline{r}_h^{-1} \cdot \vec{H}_x = \tilde{H}_x = \frac{\beta^2 - \gamma_h^2}{j\omega\mu_0} \vec{\phi}^h \qquad (5)$$

$$T_e^t \cdot \overline{r}_e^{-1} \cdot \vec{H}_z = \tilde{H}_z = \frac{-\beta\delta}{\omega\mu_0 h_0} \vec{\phi}^h - \frac{d\vec{\phi}^e}{dy}$$

and

$$\frac{d^2\vec{\phi}^{e,h}}{dy} - \gamma_{e,h}^2 \cdot \vec{\phi}^{e,h} = 0 \qquad (6)$$

Note that physical meaning of the parameters used in the above equations can refer to [3,4]. In addition, field quantities and potentials in the inhomogeneous subregions (I,II,III") are subject to the similar relationship as (5) and (6).

The solution to (6) takes the same form as (2) and hence,

$$\left.\frac{d\vec{\phi}^{e,h}}{dy} \Big/ \vec{\phi}^{e,h}\right|_{y_2} = Q^{e,h}$$

$$\frac{\gamma_{e,h} \cdot \tanh(\gamma_{e,h}(y_2 - y_1)) \cdot \vec{\phi}_{y_1}^{e,h} + \left.\frac{d\vec{\phi}^{e,h}}{dy}\right|_{y_1}}{\vec{\phi}_{y_1}^{e,h} + \frac{1}{\gamma_{e,h}} \cdot \tanh(\gamma_{e,h}(y_2 - y_1)) \cdot \left.\frac{d\vec{\phi}^{e,h}}{dy}\right|_{y_1}} \qquad (7)$$

where $Q^{e,h}$ is different from one subregion to another. From (5) and (7), a field matrix relating magnetic field quantities to electric counterpart can easily be derived at any location along y-direction in each subregion. After transformation back into the original domain, the field matrix can be read as

$$\begin{cases} H_I|_{y=-(s+w)} = G_I \cdot E_I|_{y=-(s+w)} \\ H_{III',III''}|_{y=-w} = G_{III',III''} \cdot E_{III',III''}|_{y=-w} \end{cases} \qquad (8)$$

Matching the tangential field components of different subregions at y = -(s+w) and y = -w and keeping in mind that similar field expansion and matrix manipulation as (5) and (8) can be completed for the subregion II, a characteristic matrix is obtained

$$G_0 \cdot E|_{y=-w} = 0 \qquad (9)$$

Now, frequency-dependent propagation constants can be found through the determinant equation and eigenvalue analysis.

NUMERICAL RESULTS

Fig.2 shows a comparison for a simple microstrip between our results of characteristic impedance and those in [2] obtained by the method of lines. An excellent agreement is observed when w/h is greater than 1.0 and difference between numerical results of both approaches becomes more and more important as w/h approaches 0. This phenomenon can be explained by the fact that this novel technique can well accommodate edge singularities of the strip with a consistent numerical accuracy whatever the strip width is. However, algorithm used in [2] has to be adjusted with different line width. For this reason, our results should be more accurate than [2]. As shown in Fig.3, results for the effective permittivity of a single microstrip, which are obtained by the hybrid-mode analysis based on this novel technique, agree well with the results in [2] by using the conventional method of lines. However, a similar deviation is increasingly pronounced as the width of the strip becomes more and more narrow. This can be explained by the same reason.

Fig.4 demonstrates a contour plot of calculated potential of a suspended microstrip by using the quasi-static analysis of this novel technique. Unbounded bilateral boundaries can exactly be simulated through this approach where electric potentials are not necessary to be null. However, the electric walls are generally assumed in the classical modeling. Fig.5 displays characteristic impedance as functions of structural parameters for unbounded multilayered composite microstrip lines. It shows that finite width and low dielectric material composite have a deviation of impedance at approximately 10%. Fig.6 shows dispersion characteristics of a symmetrical composite microstrip by use of the hybrid-mode analysis. It can be seen that narrower width (s = 0.127 mm) of the substrate supports a quasi-TEM mode with less dispersion than wider substrate (s = 0.254 mm).

CONCLUSION

An efficient numerical technique for analysis of complex microwave circuits is presented. This approach, which can be conceived as the counterpart of the spectral domain mode-matching method, is able to significantly reduce the edge singularity and hence improve greatly the numerical accuracy in general. This is in particular useful in the monolithic (hybrid) microwave integrated circuits (M(H)MIC) where the line dimensions become extremely miniature. In particular, geometries with a combination of very narrow strips/very wide slots or vice-versa can easily be handled with consistent accuracy. Bilaterally unbounded structures with segmented multilayers can accurately be simulated. Numerical results of planar circuits with composite multilayers demonstrate the versatility and effectiveness of this novel method.

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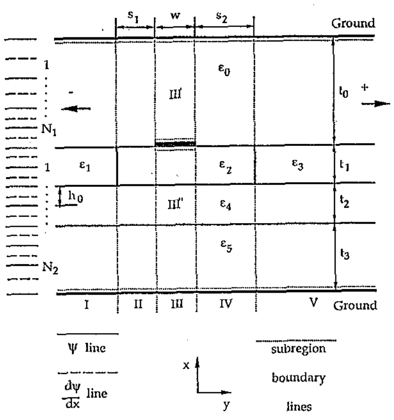

Fig.1: Illustration of a unbounded multilayer planar transmission line with a composite layer. The whole structure is segmented into six subregions.

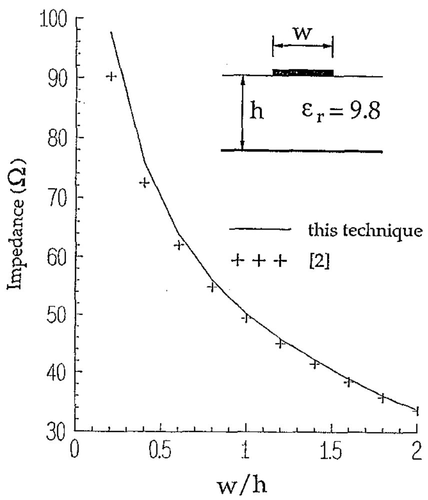

Fig.2: Comparison of characteristic impedance as a function of the ratio w/h for a microstrip line between this technique and the conventional method of lines [2].

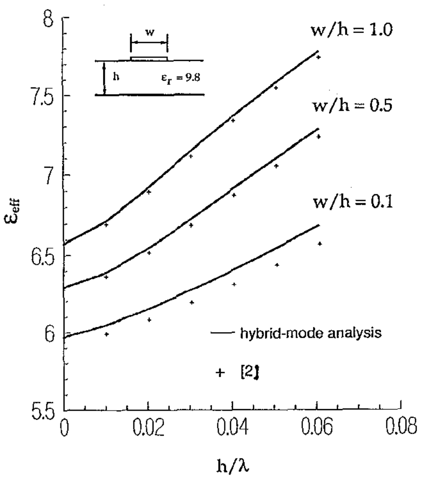

Fig.3: Dispersion characteristics of a microstrip line as a function of h/λ with different ratio w/h from 0.1 to 1.

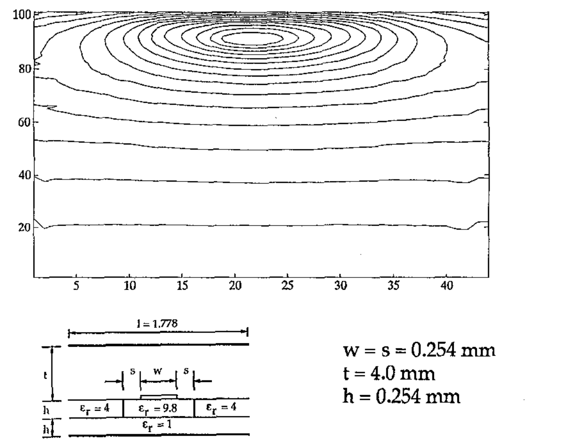

Fig.4: Contour plot of quasi-static electric potentials for a composite unbounded two-layer microstrip line. The integers used in the figure are the number of discrete points with equal intervals of 0.04508 mm along x-direction and 0.03951 mm along y-direction.

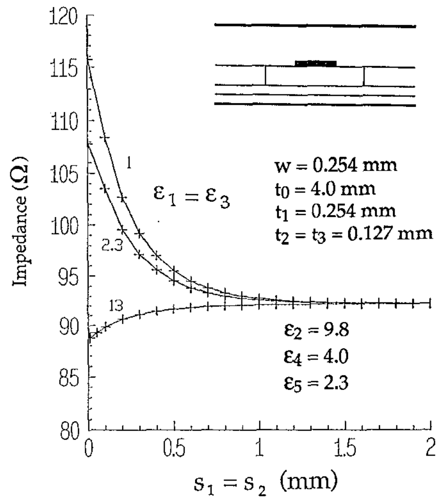

Fig.5: Impedance characteristics of a unbounded multilayered composite microstrip line as a function of s_1 (= s_2) with different outer dielectric.

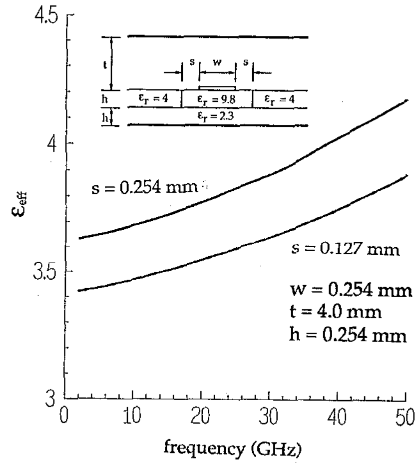

Fig.6: Frequency-dependent effective dielectric constant of a composite unbounded microstrip line versus two different s.